Section 6

Consecutive Integers

You will always see the word "**consecutive**" in a Consecutive Integers problem. Consecutive means "directly following", so a Consecutive Integer means the next whole number. For example, if you start with the number "6", the next Consecutive Integer would be "7".

Step 1
Read Through The Entire Problem

It is extremely important to carefully read through a Consecutive Integers Problem, because there are two important facts you need to determine before moving on to the next step:
- Will you be using consecutive integers, consecutive *odd* integers, or consecutive *even* integers?
- How many integers you will be using in the equation?

? ? ? ? ? ? ? ? ? ?

Step 2
Name The Expressions For The Integers

There are two ways to set up the expressions for consecutive integers. This is determined by whether the problem is about plain consecutive integers, or whether it is about either consecutive *odd* or *even* integers.

The setup will begin with an unknown, expressed by the variable x. That variable *always* represents the first integer. In a problem dealing with plain consecutive integers, the second integer would be the number right after x, which would make it $x + 1$. The third integer would be one more than the second, making it $x + 1 + 1$, or $x + 2$.

In problems dealing with consecutive odd or even integers, the variable x still represents the first integer. But because we are working with either odd or even numbers, we are skipping over a number to get to the next desired integer.

Therefore, in a problem dealing with consecutive odd or even integers, the second integer would be 2 numbers after the x, which would make it $x + 2$. The third integer would be two more than the second integer, making it $x + 2 + 2$ or $x + 4$.

Helpful Hint

- In all Consecutive Integer Problems, the **first integer** may also be referred to as the **smaller or smallest**.
- The **second integer** is referred to as the **middle**, except if there are only two integers in the problem. *(Then the second integer would be the larger or largest)*.
- The **third integer** will also be referred to as the **larger or largest**.

Step 3
Set Up An Equation

With Consecutive Integers Problems, there are two different ways to set up the equation.
- If the word "sum" is stated in the problem, add up the expressions for the integers you set up in Step 2 and set them equal to the given sum.
- If you don't see the word "sum", you will see other Direct Translation Words. In that case, you use Direct Translation in order to set up an equation. (Review Section 1 if necessary.)

Step 4
Solve the Equation

Using the method taught by your instructor, solve the equation for the variable.

Step 5
Make Sure to Answer the Question Being Asked

As in previous Chapters, when you solve the equation you will find the value for x, but that might not be the answer to the question. You need to reread the problem and make sure exactly what question is being asked.

It is possible that the value for the variable x may be your answer. But it may *not* be.

For example, the value for x will always give you the *first* integer, while the problem may be asking for the *second* or the *largest* integer. In order to get the correct answer, you will have to substitute the value for x into the original expression for the desired integer that you will set up in Step 2. Always be sure of exactly what the question is.

CONSECUTIVE INTEGER TABLE

Integer	Setup for Consecutive Integers	Setup for Odd or Even Consecutive Integers	EXAMPLE 1st Integer is 5	EXAMPLE 1st Integer is 11 (odd integer)	EXAMPLE 1st Integer is 8 (even integer)
First	x	x	5	11	8
Second	$x + 1$	$x + 2$	$5 + 1 = 6$	$11 + 2 = 13$	$8 + 2 = 10$

Third	$x + 2$	$x + 4$	$5 + 2 = 7$	$11 + 4 = 15$	$8 + 4 = 12$

EXAMPLES

EXAMPLE 1   The sum of three consecutive integers is 72. Find the three integers.

SOLUTION

Step 1 *Read The Problem*

- Seeing the word "consecutive" verifies that it is a *plain* Consecutive Integers problem.
- The problem states "consecutive integers". There is no mention of Odd or Even.
- The problem states that there are *three* consecutive integers.

Step 2 *Name The Expressions*

- The first integer is x.
- These are plain consecutive integers, so the second integer is $x + 1$.
- The third integer is $x + 2$.

$$
\begin{array}{ll}
\text{First} & = x \\
\text{Second} & = x + 1 \\
\text{Third} & = x + 2
\end{array}
$$

Step 3 *Set Up The Equation*

- The word "sum" is used in the example.
- Add the expressions you named for the integers in step 2.
- Set the expressions equal to the total (72) which is given.

$$
x + x + 1 + x + 2 \ = \ 72
$$

Step 4 *Solve The Equation*

•The solution to the equation is

$$x = 23$$

Step 5 *Answer The Question Asked*

•You have the solution to the equation, but it is NOT the answer to the question.
•The value of x is the 1st integer; but you need to find all three integers.
•You need to use the expressions for the 2nd and 3rd integers that you named in Step 2.
•Get the answer by substituting the solution for x (which is 23) into the expressions.
•You may use common sense to get the 2nd and 3rd integers (Step 5 Helpful Hint).

2nd integer $= x + 1$ 3rd integer $= x + 2$
2nd integer $= 23 + 1$ 3rd integer $= 23 + 2$ *Answer*: The 3 integers are 23, 24, 25
2nd integer $= 24$ 3rd integer $= 25$

EXAMPLE 2   The sum of three consecutive odd integers is 63. Find the largest of the three integers.

SOLUTION

Step 1 *Read The Problem*

- Seeing the word "consecutive" verifies that it is a Consecutive Integers problem.
- The problem states "consecutive *odd* integers".
- The problem states that there are three consecutive integers.

Step 2 *Name The Expressions*

- The first integer is x.
- These are consecutive odd integers, so the second integer is $x + 2$.
- The third integer is $x + 4$.

$$
\begin{aligned}
\text{First} &= x \\
\text{Second} &= x + 2 \\
\text{Third} &= x + 4
\end{aligned}
$$

Step 3 *Set Up The Equation*

- The word "sum" is used in the example.
- Add the expressions you named for the integers in step 2.
- Set the expressions equal to the total (63) which is given.

$$x + x + 2 + x + 4 = 63$$

Step 4 *Solve The Equation*

- The solution to the equation is

$$x = 19$$

Step 5 *Answer The Question Asked*

•You have the solution to the equation, but it is NOT the answer to the question.
•The value of x is the 1ˢᵗ (smallest) integer, but you need to find the largest integer.
•You need to use the expressions for the largest (3ʳᵈ) integer that you named in Step 2.
•Get the answer by substituting the solution for x (which is 19) into the expression.
•You may use common sense to get the largest integer (Step 5 Helpful Hint).

$$\text{Largest Integer} = x + 4$$
$$\text{Largest Integer} = 19 + 4$$
$$\text{Largest Integer} = 23$$

Answer: The largest integer is 23.

EXAMPLE 3   Find two consecutive integers such that the larger is 8 less than twice the smaller.

SOLUTION

Step 1 *Read The Problem*

- Seeing the word "consecutive" verifies that it is a Consecutive Integers problem.
- The problem states "consecutive *even* integers".
- The problem states that there are *two* consecutive integers.
- There are Translation Words [less than, is, twice].

Step 2 *Name The Expressions*

- The first (smaller) integer is x.
- There are only two integers so the second integer is the larger integer.
- These are consecutive even integers so the second (larger) integer is $x + 2$.

$$
\begin{array}{l}
\text{First (smaller)} \ = x \\[4pt]
\text{Second (larger)} = x + 2
\end{array}
$$

Step 3 *Set Up The Equation*

- You do NOT see the world "sum". You DO have Direct Translation Words.
- Use the expressions you named for the integers in Step 2.
- Set up an equation using Direct Translation. (Review Section 1 if necessary.)
- The larger integer will be set equal to 8 less than twice the smaller.

$$
x + 2 = 2x - 8
$$

Step 4 *Solve The Equation*

- The solution to the equation is

$$
x = 10
$$

Step 5 *Answer The Question Asked*

    •You have the solution to the equation, but it is NOT the answer to the question.

    •The value of x is the 1st (smallest) integer, but you need to find both integers.

    •You need to use the expressions for the second (larger) integer that you named in Step 2.

    •Get the answer by substituting the solution for x (which is 10) into the expression.

    •You may use the common sense approach to get the larger integer (Step 5 Helpful Hint).

$$\text{Larger integer} = x + 2$$
$$\text{Larger integer} = 10 + 2$$
$$\text{Larger integer} = 12$$

Answer: The larger integer is 12.

Consecutive Integers: Exercise Set

1. The sum of two consecutive integers is 41. Find the integers.

2. The sum of two consecutive integers is 193. Find the integers.

3. The sum of two consecutive odd integers is 32. Find the integers.

4. The sum of two consecutive odd integers is 104. Find the integers.

5. The sum of two consecutive even integers is 66. Find the integers.

6. The sum of two consecutive even integers is 90. Find the integers.

7. The sum of three consecutive integers is 93. Find the integers.

8. The sum of three consecutive integers is 120. Find the integers.

9. The sum of three consecutive odd integers is 75. Find the integers.

10. The sum of three consecutive odd integers is 153. Find the integers.

11. The sum of three consecutive even integers is 72. Find the integers.

12. The sum of three consecutive even integers is 138. Find the integers.

13. Find the second of two consecutive integers if the second is 13 less than twice the first.

14. Find the larger of two consecutive integers if the larger is 149 less than twice the smaller.

15. Find the smaller of two consecutive odd integers if the larger is 20 less than three times the smaller.

16. Find the larger of two consecutive odd integers if the larger is 40 less than three times the smaller.

17. Find the larger of two consecutive even integers if twice the smaller is 48 more than the larger.

18. Find the smaller of two consecutive even integers if three times the smaller is 8 more than twice the larger.

19. Find the second of three consecutive integers if the sum of twice the first and 4 times the second is equal to 20 more than twice the third.

20. Find three consecutive integers if the third is equal to 15 less than the sum of the first and second.